MODELING IN PHYSICS AND MATH`s EDUCATION

Moscow
2014

Moscow State Pedagogical University

Institute of Physics, Technology and Information Systems
Faculty of Physics and Information Technologies

University of Cologne

Faculty of Mathematics and Natural Sciences

Mathematical Institute

Modeling in Physics and Math's Education

The materials of Russian–German Seminar
in Moscow – Cologne, 2013

Moscow - 2014

УДК 537.8
ББК 22.38

Modeling in Physics and Math's Education. The materials of Russian –German Seminar in Moscow – Cologne, 2013.- Moscow: MPSU, 2014. - 100 p.

ISBN 978-5-4263-0200-6

The collection includes materials of the Russian–German seminar which took place from September, 28, to October, 8, 2013 in Moscow (Russia) at the Faculty of Physics and Information Technologies of Moscow State Pedagogical University and from November, 27, to December, 5, 2013 in Cologne (Germany) at the Mathematical Institute of University of Cologne.

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ISBN 978-5-4263-0200-6

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EXPERIENCE IN MATHEMATICAL MODELING AS PARTE OF TEACHERS' TRAINING

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The esteem for Mathematics in general and for the subject at school in particular does not reflect the importance which it deserves due to its role as one of the fundamental fields of knowledge. Typically the occurrence of Mathematics in discussions seems to polarize: The reaction ranges either from admiration to abrupt refusal - both expressed usually without any true knowledge.

Certainly that situation is unsatisfactory since it does not at all reflect the fascination which stems from the beauty of Mathematics and its clear logical theories which have lead to the title as "Queen of Science" nor does it acknowledge the manifold applications our modern technically based society relies on in most advanced technologies as well as in every day encounters.

What is the reason for such divergent views?

Notwithstanding the practical need to care for a solid education in Mathematics and to promote the desire to choose Mathematics as a promising subject for a professional training it is the duty of Mathematicians to raise sympathy for their field.

One reason for the lack of understanding may be seen in a specific feature of Mathematics aiming at abstraction. Certainly this is an essential part of its power but at a first view and for an untrained observer this peculiarity is hard to grasp. Following the principle of abstraction allows to build up a general language which is able to express diverse situations in such a way that common features become visible, inherent (hidden) mechanisms are detected and paths of interactions are shown in an easily understandable way. Hence, while abstraction works as a true power of Mathematics it is exactly that approach which is so difficult to follow for many people and pupils in particular (4).

The elegant way of presenting Mathematics as derived from basic axioms and demonstrating various applications to diverse situations as a natural outcome often comes as a surprise and bears the danger to hide the stepwise development and to let Mathematics appear as a "secret" science accessible only to very few especially gifted.

While an abstract and to a large extent generalized formulation provides an elegant and efficient way to present mathematical results for the experienced it does not appear as an appropriate approach for beginners who need more motivation and a rather elementary introduction to understand the key ideas and the essentials of the method. Hence to work out new ideas it is better to start with examples as easy as possible but still containing the key elements of the problem and to address the possibility to generalize in a systematic way later on. For additional motivation it is always helpful to connect the problem with situations familiar from daily experience if possible or to build up experience by examples where mathematics occurs in a surprising often hidden way.

There are various ways to popularize Mathematics and to strengthen the public attention to value the importance of Mathematics (2, 5, 7). As a successful action drawing great attention the many public demonstrations during the "Year of Mathematics" (2008) can be seen. In addition to such occasional events there is a need for lasting procedures. To achieve that goal the benefits and the pleasure of Mathematics should be taught at an early stage using examples from recreational mathematics or familiar cases within elementary experience exhibiting a mathematical background. To prepare such lessons requires experience and skill, and for that reason it is desirable to include such processes into teachers` training.

At the University of Cologne that idea has been realized in a series of seminars especially offered to students aiming at a degree as teachers (9, 14). Key elements of these seminars were related to situations taken from experience already accessible to pupils: for example simple mechanical experiments such as the pendulum with its diverse variations, toys based on phenomena with a mathematical background or the

fundamental role of algorithms prevalent in many applications. The outcome of some these seminars have been published in (13).

As typical examples related to simple mechanical objects often transformed into toys we have used the following specimen:

Wooden Toys based on impacts and friction ("Tumbling toy, tumbling duck, woodpecker, Celt or rattlebeck), various forms of tippe tops, rotating disks or wheels. Besides the playful aspects of such objects there is very often a weighty contraption useful in more serious applications. Another rich source of suitable topics can be found in the combination of Mathematics and Sports (2, 10, 11, 12, 13, 15).

A fruitful approach to raise interest in Mathematics at an early stage has been derived in form of special lectures held at the annual "Kinderuni" organized at many universities and at the University of Cologne since 2003 with the idea to get young children interested in scientific subjects. The range of subjects taught to children at the age of 8 to 12 years contains almost all areas in science, medicine or the humanities. To present scientific results in such a way which is comprehensible by children without neglecting academic claims is a great challenge.

Rather than adding just knowledge it is the primary goal of the those lectures to get children interested into research, to provide insight into subject, methods or results of scientific work. Starting with "elementary" questions can often be used as a successful introduction. While experimental subjects or medicine often provide a natural access a more abstract topic such as Mathematics requires special considerations. Nevertheless there are plenty of suitable subjects, and in the following we will briefly review some examples taken from the curriculum at the University of Cologne over the past decade; a detailed description of these lectures has been edited in the thesis by Kohlhase (7).

The common feature of these lectures relies on the facts that the topics are chosen from a range the children are familiar with, often with connection to local specialities. In addition to this the subject should serve as a way to give deeper insight into mathematics by giving an impression of mathematical thinking. In the following we will provide a few examples.

“When the Famous Emperor`s Bell in the Cathedral of Cologne Refused To Ring and How Mathematicians Helped Out”

The construction of the famous cathedral of Cologne was started in 1248 but was not completed before the end of the nineteenth century when the great building became a symbol of arising German nationality. When it was finished the citizen of Cologne had the desire to crown that impressive construction by a large bell – at that time the largest free swinging bell of the world. Since there was no experience with such huge objects the casting turned out to be difficult. When eventually successfully finished it turned out that the bell did not ring at all. There was no proper sound produced because the clapper never hit the wall of the bell but rather stayed in the middle of the bell. For a bell of that size it was difficult to change the constellation, and an intense public discussion started, partly with humorous or sarcastic contributions. Soon after the failure, a mathematics teacher from a town nearby provided a convincing mathematical analysis based on the model of a double pendulum offering ways to improve the bell (8, 17, 18).

The cathedral of Cologne is an object familiar to the children, the story of the bell is accompanied by entertaining contributions; although relying on differential equations the analysis can roughly be explained to the children, and using a model it is possible to carry out experiments illustrating the mathematical results. Furthermore that specific example offers many possibilities to present interesting results about oscillations and the dynamics of systems including for example chaotic motions.

Three Times Zero Is zero Is zero Is …What Is Peculiar about Zero?

There is a very popular carnival song referring to the very simple mathematical fact with an ironical remark onto the basic rules of multiplication taught at elementary schools. Multiplying with zero is very easy but usually nobody takes the special feature of zero into account. Historically there is a twofold meaning of zero: as the value on one hand and on the other hand as its function to mark an empty space. The inherent dramatic feature of zero has been employed by Fritz Lang who created the nowadays usual countdown for his movie “The journey to the moon“ a

long time before the first real rockets started. There are plenty of interesting stories around zero and its strange properties ranging from philosophical considerations to the failure of engines in large American warship “Yorktown” in 1977 due to a careless computer program not preventing division by zero.

Isn’t That Strange? Mathematical Paradoxes!

Usually mathematics is considered as a strict logical science without any double meanings. On the other hand many mathematical concepts have been developed in the procedure to remove gaps or incompleteness. For example the square root of 2 had a meaning to the Greeks as a geometrical object, as the length of the diagonal of the unit square, but not as a number, hence leading to the introduction of irrational numbers. There are many more paradoxes known in Mathematics or logic, puzzles, geometrical illusions ranging from historic examples up to questions in modern Mathematics as the Banach-Tarski paradox.

Knowing Math Helps To Gain! Strategies for Mathematical Games

Many games rely on mathematical structures and of course it helps to win if the structures are detected and if an appropriate strategy will be applied. The list of games includes counting verses, take away games and games involving dices relying on stochastic properties. A surprising example is given by the Efron dices referring to a cyclic process.

Cryptology and Encoding

Cryptology and cipher codes are a fascinating subject for children occurring in many detective or spy stories but codes are also used in many common procedures as pin codes or strip codes or in telecommunication. The illustration of historic technics mixed with interesting stories offers plenty possibilities to show mathematical approaches and to develop personal skills in coding.

Mazes and Labyrinths

Mazes and labyrinths already occur in ancient stories. The famous "Cretan Labyrinth" attributed to Daedalos was created to host the Minotaur. Later on mazes formed by hedges became part of pompous gardens. They also have cut a great figure in recreational mathematics (2). From a more abstract view they can be considered as graphs or networks. Simple tasks as drawing the graph known as the "Haus des Nikolaus" without following a piece twice are familiar to every child; the sophisticated version due to Euler is known as the problem of the bridges in Königsberg. Starting from such elementary examples it is possible to provide an introduction into the theory of networks referring as well to modern applications.

Sun, Coins and Wheels: Math around the Circle

Circles are everywhere. Although easy to construct the converse is more difficult to find out. How to decide if a hole has a circular shape? This somehow artificially sounding question nevertheless touches a serious topic: in quality control it is often necessary to check if objects satisfy certain norms, for example if they are truly circular. Surprisingly for many people measuring the diameters does not suffice. There are many objects with constant diameters which are not circular. They can be found in (English) coins, engines (Wankel), strange tools used to drill (almost) square holes, hydrants in New York (not accessible to usual tools) or as funny wheels of a strange looking vehicle. Sometimes circles appear surprisingly as sun spots in a forest where they are not supposed to. There is plenty of Mathematics around circles and the magic number Pi which can be illustrated at a traditional Kölsch glass used in Cologne to serve the local beer (1).

Strange Bands and Entangled Knots – Math and Magic

The Möbius strip is a simple object with surprising mathematical properties which have been illustrated in art works as well. Starting with it surprising effects of such bands the underlying mathematical ideas are developed and illustrated by some magic tricks relying on topological properties.